

انتقال حرارت هدایت پیشرفته
(conduction)

حل یک سری از معادلات دیفرانسیل با استفاده از متدهای کلوکیشن و حداقل مربعات (CM and LSM)
با استفاده از نرم افزار MAPLE

DO NOT COPY (www.novin-eng.ir)

تمرین ۱:

$$\frac{dT(t)}{dt} + T(t) = 0 \quad \text{for } t > 0$$

$$\text{IC: } T(t=0) = 1$$

حل دقیق $T(t) = e^{-t}$

روش collocation

> restart;

$$> T(t) := c[0] + c[1] \cdot t + c[2] \cdot t^2 + c[3] \cdot t^3 + c[4] \cdot t^4 + c[5] \cdot t^5 + c[6] \cdot t^6 + c[7] \cdot t^7 + c[8] \cdot t^8 \\ + c[9] \cdot t^9 + c[10] \cdot t^{10} + c[11] \cdot t^{11} + c[12] \cdot t^{12} + c[13] \cdot t^{13};$$

$$T := t \rightarrow c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 + c_5 t^5 + c_6 t^6 + c_7 t^7 + c_8 t^8 + c_9 t^9 + c_{10} t^{10} + c_{11} t^{11} \\ + c_{12} t^{12} + c_{13} t^{13}$$

> eq := subs(T(0) = 1, T(t));

$$eq := t^{13} c_{13} + t^{12} c_{12} + t^{11} c_{11} + t^{10} c_{10} + t^9 c_9 + t^8 c_8 + t^7 c_7 + t^6 c_6 + t^5 c_5 + t^4 c_4 + t^3 c_3 \\ + t^2 c_2 + t c_1 + 1$$

> eq1 := diff(eq, t) + eq;

$$eq1 := t^{13} c_{13} + t^{12} c_{12} + 13 t^{12} c_{13} + t^{11} c_{11} + 12 t^{11} c_{12} + t^{10} c_{10} + 11 t^{10} c_{11} + t^9 c_9 \\ + 10 t^9 c_{10} + t^8 c_8 + 9 t^8 c_9 + t^7 c_7 + 8 t^7 c_8 + t^6 c_6 + 7 t^6 c_7 + t^5 c_5 + 6 t^5 c_6 + t^4 c_4 \\ + 5 t^4 c_5 + t^3 c_3 + 4 t^3 c_4 + t^2 c_2 + 3 t^2 c_3 + t c_1 + 2 t c_2 + c_1 + 1$$

> R[1/14] := subs(t = 1/14, eq1);

$$R_{\frac{1}{14}} := \frac{15}{14} c_1 + \frac{29}{196} c_2 + \frac{43}{2744} c_3 + \frac{57}{38416} c_4 + \frac{71}{537824} c_5 + \frac{85}{7529536} c_6 \\ + \frac{99}{105413504} c_7 + \frac{113}{1475789056} c_8 + \frac{127}{20661046784} c_9 + \frac{141}{289254654976} c_{10} \\ + \frac{155}{4049565169664} c_{11} + \frac{169}{56693912375296} c_{12} + \frac{183}{793714773254144} c_{13} + 1$$

> R[2/14] := subs(t = 2/14, eq1);

> R[3/14] := subs(t = 3/14, eq1);

> R[4/14] := subs(t = 4/14, eq1);

```

> R[ 5/14 ] := subs( t = 5/14, eq1 ) :
> R[ 6/14 ] := subs( t = 6/14, eq1 ) :
> R[ 7/14 ] := subs( t = 7/14, eq1 ) :
> R[ 8/14 ] := subs( t = 8/14, eq1 ) :
> R[ 9/14 ] := subs( t = 9/14, eq1 ) :
> R[ 10/14 ] := subs( t = 10/14, eq1 ) :
> R[ 11/14 ] := subs( t = 11/14, eq1 ) :
> R[ 12/14 ] := subs( t = 12/14, eq1 ) :
> R[ 13/14 ] := subs( t = 13/14, eq1 ) :
> fsolve( { R[ 1/14 ], R[ 2/14 ], R[ 3/14 ], R[ 4/14 ], R[ 5/14 ], R[ 6/14 ], R[ 7/14 ], R[ 8/14 ], R[ 9/14 ],
            R[ 10/14 ], R[ 11/14 ], R[ 12/14 ], R[ 13/14 ] } );
            { c1 = -0.9999994619, c2 = 0.4999880587, c3 = -0.1665220033, c4 = 0.04056377582, c5 =
              -0.002677790842, c6 = -0.01886508894, c7 = 0.05147394927, c8 = -0.09445241021, c9
              = 0.1228630224, c10 = -0.1109036889, c11 = 0.06601036485, c12 = -0.02328317469, c13
              = 0.0036839171 }

```

```

> eq2 := subs( c1 = -0.9999994619, c2 = 0.4999880587, c3 = -0.1665220033, c4
              = 0.04056377582, c5 = -0.002677790842, c6 = -0.01886508894, c7 = 0.05147394927, c8
              = -0.09445241021, c9 = 0.1228630224, c10 = -0.1109036889, c11 = 0.06601036485, c12
              = -0.02328317469, c13 = 0.0036839171, eq );

```

$$\begin{aligned}
 eq2 := & 0.0036839171 t^{13} - 0.02328317469 t^{12} + 0.06601036485 t^{11} - 0.1109036889 t^{10} \\
 & + 0.1228630224 t^9 - 0.09445241021 t^8 + 0.05147394927 t^7 - 0.01886508894 t^6 \\
 & - 0.002677790842 t^5 + 0.04056377582 t^4 - 0.1665220033 t^3 + 0.4999880587 t^2 \\
 & - 0.9999994619 t + 1
 \end{aligned}$$

```

> eq3 := exp(-t);

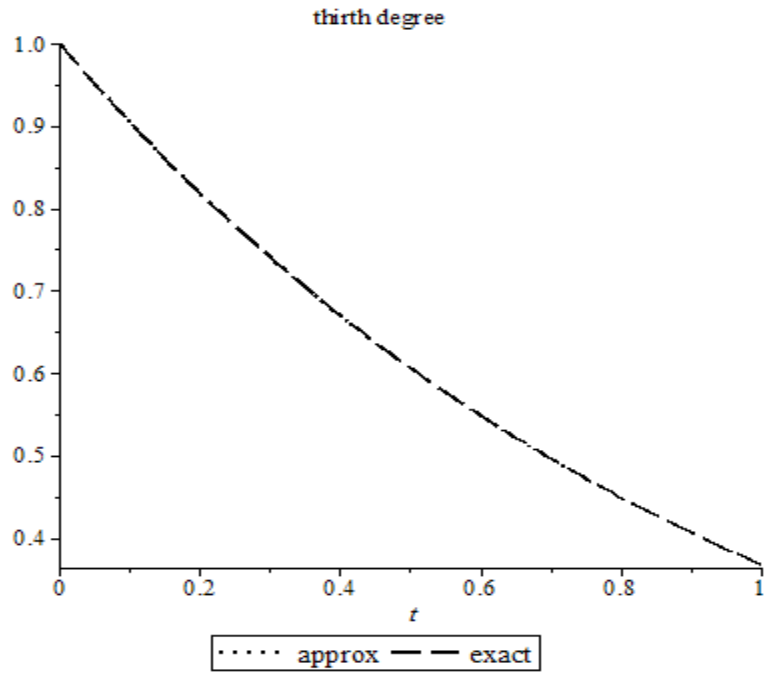
```

$$eq3 := e^{-t}$$

```

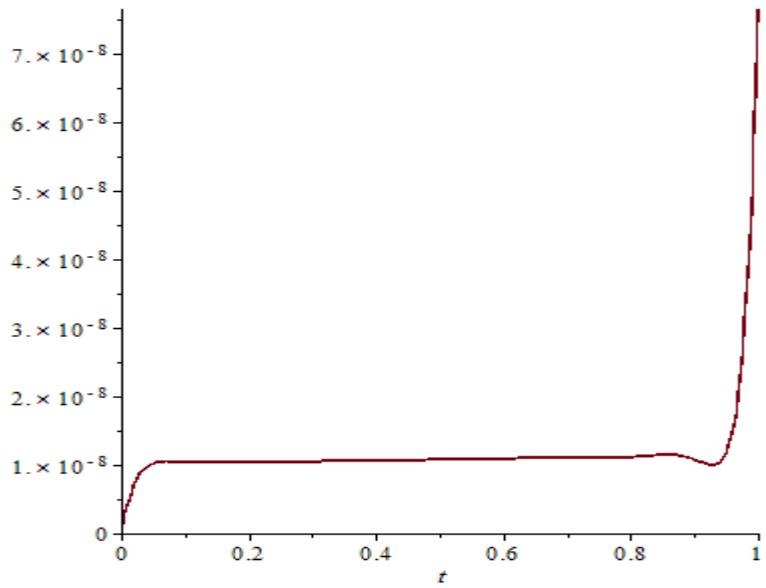
> plot([eq2, eq3], t = 0..1, title = "third degree", color = [black], linestyle = [DOT, DASH],
       legend = ["approx", "exact"]);

```



> error := $\frac{eq2 - eq3}{eq3}$;

> plot(error, t = 0..1);



تمرین ۲:

$$\frac{dT}{dx} + T^4 = 0$$

$$T(x=0)=1$$

روش C-M

>restart;

$$\begin{aligned} > T(t) := & c[0] + c[1] \cdot t + c[2] \cdot t^2 + c[3] \cdot t^3 + c[4] \cdot t^4 + c[5] \cdot t^5 + c[6] \cdot t^6 + c[7] \cdot t^7 + c[8] \cdot t^8 \\ & + c[9] \cdot t^9 + c[10] \cdot t^{10} + c[11] \cdot t^{11} + c[12] \cdot t^{12} + c[13] \cdot t^{13}; \end{aligned}$$

$$\begin{aligned} T := & t \rightarrow c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 + c_5 t^5 + c_6 t^6 + c_7 t^7 + c_8 t^8 + c_9 t^9 + c_{10} t^{10} + c_{11} t^{11} \\ & + c_{12} t^{12} + c_{13} t^{13} \end{aligned}$$

$$> eq := \text{subs}(T(0) = 1, T(t));$$

$$\begin{aligned} eq := & t^{13} c_{13} + t^{12} c_{12} + t^{11} c_{11} + t^{10} c_{10} + t^9 c_9 + t^8 c_8 + t^7 c_7 + t^6 c_6 + t^5 c_5 + t^4 c_4 + t^3 c_3 \\ & + t^2 c_2 + t c_1 + 1 \end{aligned}$$

$$> eq1 := \text{diff}(eq, t) + eq^4;$$

$$\begin{aligned} eq1 := & 13 t^{12} c_{13} + 12 t^{11} c_{12} + 11 t^{10} c_{11} + 10 t^9 c_{10} + 9 t^8 c_9 + 8 t^7 c_8 + 7 t^6 c_7 + 6 t^5 c_6 \\ & + 5 t^4 c_5 + 4 t^3 c_4 + 3 t^2 c_3 + 2 t c_2 + c_1 + (t^{13} c_{13} + t^{12} c_{12} + t^{11} c_{11} + t^{10} c_{10} \\ & + t^9 c_9 + t^8 c_8 + t^7 c_7 + t^6 c_6 + t^5 c_5 + t^4 c_4 + t^3 c_3 + t^2 c_2 + t c_1 + 1)^4 \end{aligned}$$

$$> R\left[\frac{1}{14}\right] := \text{subs}\left(t = \frac{1}{14}, eq1\right);$$

$$> R\left[\frac{2}{14}\right] := \text{subs}\left(t = \frac{2}{14}, eq1\right);$$

$$> R\left[\frac{3}{14}\right] := \text{subs}\left(t = \frac{3}{14}, eq1\right);$$

$$> R\left[\frac{4}{14}\right] := \text{subs}\left(t = \frac{4}{14}, eq1\right);$$

$$> R\left[\frac{5}{14}\right] := \text{subs}\left(t = \frac{5}{14}, eq1\right);$$

$$> R\left[\frac{6}{14}\right] := \text{subs}\left(t = \frac{6}{14}, eq1\right);$$

$$> R\left[\frac{7}{14}\right] := \text{subs}\left(t = \frac{7}{14}, eq1\right);$$

$$> R\left[\frac{8}{14}\right] := \text{subs}\left(t = \frac{8}{14}, eq1\right);$$

$$> R\left[\frac{9}{14}\right] := \text{subs}\left(t = \frac{9}{14}, eq1\right);$$

```

> R[10/14] := subs(t = 10/14, eq1) :
> R[11/14] := subs(t = 11/14, eq1) :
> R[12/14] := subs(t = 12/14, eq1) :
> R[13/14] := subs(t = 13/14, eq1) :
> fsolve( {R[1/14], R[2/14], R[3/14], R[4/14], R[5/14], R[6/14], R[7/14], R[8/14], R[9/14],
R[10/14], R[11/14], R[12/14], R[13/14]} );
{c1 = -0.9996770709, c2 = 1.991723351, c3 = -4.550399921, c4 = 10.59009885, c5 =
-23.17246357, c6 = 44.54920496, c7 = -71.26810174, c8 = 90.83254830, c9 =
-88.64444543, c10 = 63.31056104, c11 = -31.01075237, c12 = 9.276113288, c13 =
-1.274450696}

```

```

> eq2 := subs(c1 = -0.9996770709, c2 = 1.991723351, c3 = -4.550399921, c4 = 10.59009885, c5 =
-23.17246357, c6 = 44.54920496, c7 = -71.26810174, c8 = 90.83254830, c9 =
-88.64444543, c10 = 63.31056104, c11 = -31.01075237, c12 = 9.276113288, c13 =
-1.274450696, eq);

```

$$\begin{aligned}
 eq2 := & -1.274450696t^{13} + 9.276113288t^{12} - 31.01075237t^{11} + 63.31056104t^{10} \\
 & - 88.64444543t^9 + 90.83254830t^8 - 71.26810174t^7 + 44.54920496t^6 \\
 & - 23.17246357t^5 + 10.59009885t^4 - 4.550399921t^3 + 1.991723351t^2 \\
 & - 0.9996770709t + 1
 \end{aligned}$$

```

> eq3 := (1/(3*t+1))^(1/3);
eq3 := (1/(3*t+1))^(1/3)

```

```

> plot([eq2, eq3], t = 0..1, title = "third degree", color = [black], linestyle = [DOT, DASH],
legend = ["approx", "exact"]);

```