دانشگاه صنعتی نوشیروانی بابل

ارائه انتقال حرارت هدایت پیشرفته

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SCIENCE DIRECT

International Communications in Heat and Mass Transfer 33 (2006) 391–400

International Communications in HEAT and MASS TRANSFER

www.elsevier.com/locate/ichmt

Assessment of homotopy–perturbation and perturbation methods in heat radiation equations ☆

D.D. Ganji *, A. Rajabi

Mazandaran University, Department of Mechanical Engineering, P.O. Box 484, Babol, Iran

Available online 28 November 2005

سرفصل مطالب:

معرفی مقاله

حل مقاله با Maple

حل مقاله با Flexpde

بررسی مقاله در حالت دوبعدی

3. The first example (unsteady nonlinear convective-radiative equation)

For example for heat transfer in a lumped system of combined convective-radiative heat transfers; the specific heat coefficient is linear with temperature as follows [13]:

$$C = C_a(1 + \beta(T - T_a)). \tag{3}$$

The cooling equation of the system with the following initial condition is as follows:

$$\rho V C \frac{dT}{dt} + hA(T - T_a) + E\sigma A\left(T^4 - T_s^4\right) = 0, \quad T(t = 0) = T_i. \tag{4}$$

To solve the equation we must do the following change of parameters:

$$\theta = \frac{T}{T_i}, \quad \theta_a = \frac{T_a}{T_i}, \quad \tau = \frac{t}{\rho V C_a / hA}, \quad \varepsilon_1 = \beta T_i, \quad \varepsilon_2 = \frac{E\sigma T_i^3}{h} \quad , \theta_s = \frac{T_s}{T_i} \tag{5}$$

After parameter change, the system heat transfer equation will result the following:

$$[1 + \varepsilon_1(\theta - \theta_a)]\frac{d\theta}{d\tau} + (\theta - \theta_a) + \varepsilon_2\left(\theta^4 - \theta_s^4\right) = 0, \quad \tau = 0 \rightarrow \theta = 1. \tag{6}$$

4. The second example (nonlinear convective-radiative conduction equation)

The second example to be studied is the one dimensional heat transfer in a straight fin with the length of L and the cross section area of A and the perimeter of p. The fin surface transfers heat through both convection and radiation. Suppose the temperature of the surrounding air is T_0 and the effective sink temperature for the radiative heat transfer is T_s.

We assume that base temperature of the fin is T_b and there is no heat transfer of the tip of the fin. It is also assumed that the convection heat transfer coefficient, h, and the emissivity coefficient of surface, E_g, are both constant while conduction coefficient, k, can be variable. The energy equation and the boundary conditions for the fin are as follows:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(k\frac{\mathrm{d}T}{\mathrm{d}x}\right)-\frac{hp}{A}(T-T_a)-\frac{E_g\sigma}{A}\left(T^4-T_s^4\right)=0 \tag{29}$$

$$x=0 \quad \rightarrow \quad \frac{\mathrm{d}T}{\mathrm{d}x}=0,\ x=L\rightarrow T=T_b. \tag{30}$$

Assuming k as a linear function of temperature, we have:

$$k=k_a(1+\beta(T-T_a)] \tag{31}$$

After making the equation dimensionless and changing parameters, we have:

$$\theta=\frac{T}{T_b},\ \theta_a=\frac{T_a}{T_b},\ \theta_s=\frac{T_s}{T_b},\ X=\frac{x}{L},\ N^2=\frac{hpL^2}{k_aA},\ \varepsilon_1=\beta T_b,\ \varepsilon_2=\frac{E_g\sigma T_b^3pL^3}{k_aA} \tag{32}$$

And substituting Eq. (32) in Eq. (29) we have:

$$\frac{\mathrm{d}}{\mathrm{d}X}\left\{[1+\varepsilon_1(\theta-\theta_a)]\frac{\mathrm{d}\theta}{\mathrm{d}X}\right\}-N^2(\theta-\theta_a)-\varepsilon_2\left(\theta^4-\theta_s^4\right)=0 \tag{33}$$

$$X=0\rightarrow\frac{\mathrm{d}\theta}{\mathrm{d}X}=0,\ X=1\rightarrow\theta=1. \tag{34}$$

حل مسئله اول به روش HPM با میپل:

$$\left[\frac{d\theta}{d\tau}+\theta\right]+\left[\varepsilon_1\theta\frac{d\theta}{d\tau}+\varepsilon_2\theta^4\right]=0$$

```
> restart;
> PDEtools[declare]((theta)(x), prime = x);
```

θ(x) will now be displayed as θ

derivatives with respect to x of functions of one variable will now be displayed with ' **(1)**

```
> parameters(epsilon[1], epsilon[2]);
```

$$parameters(\varepsilon_1, \varepsilon_2) \quad \textbf{(2)}$$

```
> H := (1 - p)·(diff(theta(x), x) + theta(x)) + p·(epsilon[1]·theta(x)·diff(theta(x), x)
      + epsilon[2]·theta(x)^4);
```

$$H := (1-p)\,(\theta'+\theta)+p\,(\varepsilon_1\,\theta\,\theta'+\varepsilon_2\,\theta^4) \quad \textbf{(3)}$$

```
> H := subs(theta(x) = theta[0](x) + p·theta[1](x) + p^2·theta[2](x), H);
```

$$H := (1-p)\left(\left(\theta_0(x)+p\,\theta_1(x)+p^2\,\theta_2(x)\right)_x+\theta_0+p\,\theta_1+p^2\,\theta_2\right)+p\left(\varepsilon_1\left(\theta_0+p\,\theta_1+p^2\,\theta_2\right)\left(\theta_0(x)+p\,\theta_1(x)+p^2\,\theta_2(x)\right)_x+\varepsilon_2\left(\theta_0+p\,\theta_1+p^2\,\theta_2\right)^4\right) \quad \textbf{(4)}$$

```
> H := collect(simplify(expand(%)), p) :
> theta_0(x) = exp(-x);
```

$$\theta_0 = e^{-x} \quad \textbf{(5)}$$

```
> Eq1 := d/dx theta_1(x) + theta_1(x) + epsilon[1]·theta_0(x)·d/dx theta_0(x) + epsilon[2]
      ·theta_0(x)^4;
```

$$Eq1 := \theta_1' + \theta_1 + \varepsilon_1\,\theta_0\,\theta_0' + \varepsilon_2\,\theta_0^4 \quad \textbf{(6)}$$

$> Eq1 := subs(\theta_0(x) = e^{-x}, Eq1);$

$$Eq1 := \theta_1' + \theta_1 + \varepsilon_1 e^{-x} (e^{-x})' + \varepsilon_2 (e^{-x})^4 \tag{7}$$

$> Ans1 := dsolve(\{Eq1, theta[1](0) = 0\});$

$$Ans1 := \theta_1 = \left(-\varepsilon_1 e^{-x} + \frac{1}{3} \varepsilon_2 e^{-3x} + \varepsilon_1 - \frac{1}{3} \varepsilon_2\right) e^{-x} \tag{8}$$

$> Eq2 := \frac{d}{dx} theta_2(x) + theta_2(x) + epsilon[1] \cdot theta_0(x) \cdot \frac{d}{dx} theta_1(x) + epsilon[1] \cdot theta_1(x) \cdot \frac{d}{dx} theta_0(x) + 4 \cdot epsilon[2] \cdot theta_1(x) \cdot theta_0(x)^3;$

$$Eq2 := \theta_2' + \theta_2 + \theta_0 \theta_1' \varepsilon_1 + \theta_0' \theta_1 \varepsilon_1 + 4 \theta_0^3 \theta_1 \varepsilon_2 \tag{9}$$

$> Eq2 := subs\left(\theta_0(x) = e^{-x}, \theta_1(x) = \left(\frac{1}{3} \varepsilon_2 e^{-3x} - \varepsilon_1 e^{-x} - \frac{1}{3} \varepsilon_2 + \varepsilon_1\right) e^{-x}, Eq2\right):$

$> Ans2 := dsolve(\{Eq2, theta[2](0) = 0\});$

$$Ans2 := \theta_2 = \left(-\frac{17}{12} e^{-4x} \varepsilon_2 \varepsilon_1 + \frac{3}{2} \varepsilon_1^2 e^{-2x} - 2 \varepsilon_1^2 e^{-x} + \frac{2}{3} \varepsilon_1 \varepsilon_2 e^{-x} + \frac{2}{9} \varepsilon_2^2 e^{-6x} + \frac{4}{3} \varepsilon_2 \varepsilon_1 e^{-3x} - \frac{4}{9} \varepsilon_2^2 e^{-3x} - \frac{7}{12} \varepsilon_2 \varepsilon_1 + \frac{1}{2} \varepsilon_1^2 + \frac{2}{9} \varepsilon_2^2\right) e^{-x} \tag{10}$$

$> theta(x) := e^{-x} + \left(\frac{1}{3} \varepsilon_2 e^{-3x} - \varepsilon_1 e^{-x} - \frac{1}{3} \varepsilon_2 + \varepsilon_1\right) e^{-x} + \left(-\frac{17}{12} e^{-4x} \varepsilon_1 \varepsilon_2 + \frac{3}{2} \varepsilon_1^2 e^{-2x} + \frac{2}{3} \varepsilon_1 \varepsilon_2 e^{-x} - 2 \varepsilon_1^2 e^{-x} + \frac{2}{9} \varepsilon_2^2 e^{-6x} - \frac{4}{9} \varepsilon_2^2 e^{-3x} + \frac{4}{3} \varepsilon_1 \varepsilon_2 e^{-3x} - \frac{7}{12} \varepsilon_1 \varepsilon_2 + \frac{1}{2} \varepsilon_1^2 + \frac{2}{9} \varepsilon_2^2\right) e^{-x}:$